INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Thesis Title	:	**Uniqueness of the Fourier transform on the Euclidean spaces and certain locally compact Lie groups**
Name of Thesis Supervisor(s)	:	**Dr. Rajesh K. Srivastava**
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SHORT ABSTRACT

Let $X(\Gamma)$ be the space of all finite Borel measure μ in $\mathbb{R}^2$ which is supported on the curve Γ and absolutely continuous with respect to the arc length of Γ. For $\Lambda \subset \mathbb{R}^2$, the pair (Γ, Λ) is called a Heisenberg uniqueness pair for $X(\Gamma)$ if any $\mu \in X(\Gamma)$ satisfies $\hat{\mu}|_\Lambda = 0$, implies $\mu = 0$.

We explored the Heisenberg uniqueness pairs corresponding to the spiral, hyperbola, circle, cross, exponential curves, and surfaces. Then, we prove a characterization of the Heisenberg uniqueness pairs corresponding to four parallel lines. We observe that the size of the determining sets Λ for $X(\Gamma)$ depends on the number of lines and their irregular distribution that further relates to a phenomenon of interlacing of the zero sets of certain trigonometric polynomials.

M. Benedicks [1] had extended the classical Paley-Wiener theorem (about uncertainty principle) to the class of integrable functions. That is, support of an integrable function f and its Fourier transform $\hat{f}$ both can not be of finite measure simultaneously.

The Fourier transform on a non-commutative group becomes a linear operator of large rank in contrast to real line where the Fourier transform has rank one and hence it can be think of function on $\mathbb{R}$.

We prove that if the group Fourier transform of certain integrable functions on the Euclidean motion groups (or Heisenberg motion group/ step two nilpotent Lie groups) is of finite rank, then the function has to identically zero. These results can be thought as an analogue to the Benedicks theorem that dealt with the uniqueness of the Fourier transform of integrable functions on the Euclidean spaces.

References

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