INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Programme of Study : Ph.D.

Thesis Title: Classifications of some Algebraically Positive, Diagonalizable and Stable Matrices with their Sign Patterns

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Thesis Submitted to the Department/ Center : Mathematics

Date of completion of Thesis Viva-Voce Exam : 29-06-2021

Key words for description of Thesis Work : Algebraic Positivity, Diagonalizability, Spectrally Arbitrary Pattern, Potential Stability

SHORT ABSTRACT

A real square matrix A is said to be algebraically positive if there exists a real polynomial f such that $f(A)$ is a positive matrix. We prove that a real square matrix is algebraically positive if and only if it commutes with a unique (upto scalar multiplication) rank one positive matrix. We also show that for a real square matrix A, if $\operatorname{adj}(A)$ is algebraically positive, then A is also algebraically positive. We characterize all tree sign pattern matrices that allow algebraic positivity, and all star and path sign pattern matrices that require algebraic positivity. We also identify all tree sign pattern matrices of order less than 6 requiring algebraic positivity.

We introduce the concept of an essential index for a tree sign pattern matrix. We observe that a tree sign pattern matrix requires singularity if and only if it has an essential index. Further, we give a result regarding column spaces of matrices in the qualitative class of a tree sign pattern matrix. We use this result to obtain a sufficient condition for sign pattern matrices whose graphs are trees to allow diagonalizability. We also characterize sign pattern matrices allowing diagonalizability, whose graphs are either star or path. Moreover, we give a necessary condition for a sign pattern matrix requiring diagonalizability, and describe all star sign pattern matrices requiring diagonalizability.

A square matrix M is said to be stable if all eigenvalues of M have negative real parts, and a sign pattern matrix A is said to be potentially stable if there exists a stable matrix in $Q(A)$. A sign pattern matrix A allows a properly signed nest if there exists $B \in Q(A)$ and a permutation matrix P such that the sign of the k-th leading principal minor of PBP^T is $(-)^k$ for all $k \in \{1, 2, \ldots, n\}$. We give some sufficient conditions for tree sign pattern matrices with all edges negative to allow a properly signed nest. In 1997, Johnson, Maybee, Olesky and van den Driessche proved that if a sign pattern matrix allows a properly signed nest, then it is potentially stable. However, the converse is not true, even for tree sign pattern matrices. We believe that the converse is true for tree sign pattern matrices with negative edges, which we propose as a conjecture. We prove that this conjecture is true for tree sign pattern matrices with negative edges of order at most 6. Further, we identify all potentially stable star and path sign pattern matrices with negative edges, and prove that the conjecture is valid for these classes. A sign pattern matrix A of order n is a spectrally arbitrary pattern if, for any given real monic polynomial $r(x)$ of degree n, there is a matrix in $Q(A)$ with characteristic polynomial $r(x)$. As a consequence of the results on potentially stable sign pattern matrices with negative edges, we describe all 5-by-5 spectrally arbitrary tree sign pattern matrices with negative edges.