



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS



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SHORT ABSTRACT

In this thesis we study classical hypergeometric series and Appell series over finite fields, and find finite field analogues of several product and summation formulas satisfied by the classical hypergeometric series. Hypergeometric functions over finite fields are known as Gaussian hypergeometric series. As an application of the product and summation formulas, we deduce several special values of $2F_1$, $3F_2$ and $4F_3$ -Gaussian hypergeometric series. Some of our special values of Gaussian hypergeometric series are evaluated at general arguments of the parameters. Recently, finite field analogues of Appell series F_1 , F_2 and F_3 are introduced and their relations with certain Gaussian hypergeometric series are established. Integral representations of F_1 , F_2 and F_3 are used while defining their finite field analogues. However, integral representations of F_4 are more complicated than the integral representations of F_1 , F_2 and F_3 . Therefore, it is not straightforward to find an appropriate finite field analogue of F_4 using its integral representations. To overcome this problem, we define finite field analogues of classical Appell series F_1 , F_2 and F_3 using purely Gauss sums, and this allows us to define a finite field analogue of the Appell series F_4 . We then establish finite field analogues of classical identities satisfied by the Appell series and hypergeometric series. As applications, we find several transformation formulas satisfied by the Gaussian hypergeometric series. For example, we express a $4F_3$ -Gaussian hypergeometric series as a sum of two $2F_1$ -Gaussian hypergeometric series. We also express $4F_3$ -Gaussian hypergeometric series as a product of two $2F_1$ -Gaussian hypergeometric series. Product formulas for Gaussian hypergeometric series have many significant applications. We find finite field analogues of certain product formulas satisfied by the classical hypergeometric series. We express product of two $2F_1$ -Gaussian hypergeometric series as $4F_3$ - and $3F_2$ -Gaussian hypergeometric series. We use properties of Gauss and Jacobi sums and our works on finite field Appell series to deduce these product formulas satisfied by the Gaussian hypergeometric series. We then use these transformations to evaluate explicitly some special values of $4F_3$ - and $3F_2$ -Gaussian hypergeometric series. By counting points on CM elliptic curves over finite fields, Ono found certain special values of $2F_1$ - and $3F_2$ -Gaussian hypergeometric series containing trivial and quadratic characters as parameters. Later, Evans and Greene found special values of certain $3F_2$ -Gaussian hypergeometric series containing arbitrary characters as parameters from where some of the values obtained by Ono follow as special cases. We show that some of the results of Evans and Greene follow from our product formulas including a finite field analogue of the classical Clausen's identity