



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

Linearization is a classical technique widely used to deal with matrix polynomial. The main purpose of the thesis is to construct and analyze strong linearizations of polynomial and rational matrices. The first part of the thesis is devoted to construction of strong linearizations of matrix polynomials including structure-preserving strong linearizations and the recovery of eigenvectors, minimal bases and minimal indices of matrix polynomials from those of the linearizations. The second part of the thesis is devoted to construction of strong linearizations of rational matrices including structure-preserving strong linearizations and the recovery of eigenvectors, minimal bases and minimal indices of rational matrices from those of the linearizations.

Fiedler pencils (FPs), generalized Fiedler pencils (GFPs), Fiedler pencils with repetition (FPRs) and generalized Fiedler pencils with repetition (GFPRs) are important family of strong linearizations of matrix polynomials which have been studied extensively over the years. It is well known that the family of GFPRs of matrix polynomials subsumes the family of FPRs and is an important source of strong linearizations, especially structure-preserving strong linearizations of structured matrix polynomials. We propose a unified framework for analysis and construction of a family of Fiedler-like pencils, which we refer to as extended GFPRs (EGFPRs), that subsumes all the known classes of Fiedler-like pencils such as FPs, GFPs, FPRs and GFPRs of matrix polynomials. We show that the unified framework allows us to construct structure-preserving strong linearizations with additional properties such as banded pencils with low bandwidth and preservation of sign characteristic in the case of Hermitian matrix

polynomials. Moreover, we describe the recovery of eigenvectors, minimal bases and minimal indices of matrix polynomials from those of the EGFRs and show that the recovery is operation-free. In particular, we describe the recovery of eigenvectors, minimal bases and minimal indices of matrix polynomials from those of the FPRs and GFRs which has been an open problem.

Rational matrices arise in many applications. Linearization of rational matrices has been introduced recently for solving rational eigenproblems. FPs, GFPs and FPRs for rational matrices have been constructed which are shown to be linearization of rational matrices. We introduce a strong linearization (referred to as Rosenbrock strong linearization) of rational matrices and show that structural indices of finite as well as infinite poles and zeros of rational matrices can be recovered from those of the strong linearizations. We show that FPs, GFPs and FPRs of rational matrices are in fact Rosenbrock strong linearizations. Also, we introduce a family of Fiedler-like pencils, which we refer to as generalized Fiedler pencils with repetition (GFRs), for rational matrices and show that the GFRs are strong linearizations. We show that the family of GFRs is an important source of structure-preserving strong linearization of rational matrices. In fact, we utilize GFRs to construct structure-preserving strong linearization of structured (symmetric, skew-symmetric, Hamiltonian, skew-Hamiltonian, Hermitian, para-Hermitian, etc.) rational matrices. We show that the Hermitian GFRs preserve the Cauchy-Maslov index of Hermitian rational matrices. We describe the recovery of eigenvectors, minimal bases and minimal indices of rational matrices from those of the GFRs and show that the recovery is operation-free. We also introduce affine spaces of strong linearization of rational matrices and describe the recovery of eigenvectors, minimal bases and minimal indices of rational matrices from those of the strong linearizations.