INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Thesis Title: ROBUST NUMERICAL METHODS FOR SINGULARLY PERTURBED PARABOLIC PDEs WITH INTERIOR AND BOUNDARY LAYERS

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SHORT ABSTRACT

This thesis provides some uniformly convergent numerical methods for solving singularly perturbed convection-diffusion problems with boundary or/and interior layers. A differential equation becomes singularly perturbed when a small parameter is multiplying with the highest-order derivative. The solutions of these types of problems exhibit thin boundary or/and interior layers when the small parameter tends to zero. Because of layer appearing in the solution, the classical numerical method on the uniform mesh may fail. To construct an uniformly convergent numerical scheme to this type of problem, one has to reduce the mesh size in comparison with the small parameter.

The main aim of this thesis is to apply, analyze and optimize ε-uniform fitted mesh methods (FMMs) for solving different types of singularly perturbed convection-diffusion problems with boundary or/and interior layers in 1D and 2D.

We begin the thesis with a brief introduction along with the objective and the motivation for solving singularly perturbed convection-diffusion problems. After that, we provide some basic definitions and define few terminologies which are used throughout the thesis. At first, parameter-uniform numerical scheme is proposed for solving singularly perturbed one-dimensional parabolic convection-diffusion initial-boundary-value problem (IBVP) with a boundary turning point at the left boundary of the domain. The proposed scheme consists of implicit-Euler scheme on the uniform mesh in the temporal direction and the hybrid scheme on the Shishkin mesh in the spatial direction. Then, for the same problem, we apply the Richardson extrapolation technique to improve the accuracy of the implicit upwind scheme on the piecewise-uniform Shishkin mesh in spatial direction and the uniform mesh in the temporal direction. Next, to solve singularly perturbed two-dimensional parabolic convection-diffusion IBVP with a boundary turning point, an alternating direction method on the uniform mesh and the upwind scheme on the Shishkin mesh are used for temporal derivative and spatial derivatives, respectively.

Afterwards, we mainly focus on singularly perturbed convection-diffusion problems with discontinuous convection coefficient(s) and source term in 1D as well as 2D. We start with a singularly perturbed two-point boundary-value problem (BVP) and IBVP with non smooth data, where the solutions of these problems exhibit a weak interior and boundary layers in the neighborhood of the point of discontinuity and the left boundary of the domain, respectively. For both the above mentioned problems, we replace the spatial derivatives by the hybrid scheme on the Shishkin mesh and to discretize the temporal derivative in the IBVP, we use implicit-Euler scheme on the uniform mesh. Next, depending on the sign of the discontinuous convection coefficients in the domain, we study the numerical solutions of different types of singularly perturbed two-dimensional elliptic convection-diffusion BVPs. We apply the upwind finite difference scheme on the Shishkin mesh to solve the model problem. Then, we consider the singularly perturbed two-dimensional parabolic convection-diffusion IBVP with discontinuous convection coefficient(s) and source term. An alternating direction method on the uniform mesh and the upwind scheme on the Shishkin mesh are used for temporal derivative and spatial derivatives in the model problem, respectively. To validate the theoretical findings, numerical experiments are performed, in the respective chapters. At the end of the thesis, we provide the possible extension of the work carried out in this thesis.