INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

The primary interest of this thesis is to provide some efficient numerical techniques for solving singularly perturbed delay parabolic initial-boundary-value problems (IBVPs) in one and two-dimensions. These types of problems are described by partial differential equations (PDEs) in which the highest-order derivative is multiplied by an arbitrarily small parameter ε and contain at least one delay term. Due to the parameter ε, the solution of such equation exhibits a thin layer where the solution varies rapidly, while away from the layer the solution varies slowly and behaves smoothly. Singularly perturbed PDEs model physical problems for which the evolution depend in the present state of the system. In contrast, singularly perturbed delay partial differential equations (DPDEs) relate an unknown function to its derivatives by the past history. Due to the layer phenomena, it is indeed a difficult task to provide ε-uniform numerical methods for singularly perturbed DPDEs, i.e., methods in which the approximated solution converges (measured in supremum norm) to the exact solution of the corresponding continuous problem independently with respect to the parameter ε. The purpose of this thesis is to apply, analyze and optimize the upwind based numerical methods on Shishkin-type meshes for solving singularly perturbed delay parabolic convection-diffusion problems. We begin the thesis with general introduction along with the objective and the motivation for solving singularly perturbed delay parabolic PDEs. Then, we give some definitions and terminologies which are used throughout the thesis. Next, we move towards the main work of the thesis. A uniformly convergent hybrid scheme is proposed and analyzed for a 1D DPDE on the piecewise-uniform Shishkin mesh. The hybrid scheme used here, is a proper combination of the midpoint upwind scheme and the central difference scheme. Then, we consider the same problem and use a post-processing technique (Richardson extrapolation), which improves the first-order accuracy of the standard upwind scheme applied on the piecewise-uniform Shishkin mesh to second-order convergence. Numerical experiments are carried out for singularly perturbed linear and semilinear DPDEs. Then we proceed towards two-dimensional parabolic PDEs. We use a fractional-step method to discretize the time-derivative of the singularly perturbed two-dimensional PDE. The resulting one-dimensional equations are solved by the classical upwind scheme along with the Richardson extrapolation technique. Next, we consider singularly perturbed two-dimensional DPDEs. We use the classical upwind scheme to discretize the spatial derivatives on the standard Shishkin mesh and the Bakhvalov-Shishkin mesh. By considering the fact that the fractional-step method allows us to reduce the computational cost, we use the fractional-step method along with the classical upwind scheme to solve singularly perturbed two-dimensional DPDE. We present numerical results to validate the theoretical findings.